

M. Math 1st/B. Math 3rd 2009-2010 Topology Midterm examination

points: 8x5=40

18-09-2009

Time: 3hrs

Show complete work. You may assume any results proved in the class, but need to quote them correctly. Each question is worth 5 points.

1) Let  $X$  be a topological space. Formulate and prove the statement 'subnet of a net is a net'. Show that if a net converges to a point, subnet of the net converges to the same point.

2) Let  $X$  be an infinite set with a regular topology  $\tau$  on it. Show that there exists a sequence  $\{x_n\}_{n \geq 1}$  and a sequence of pair-wise disjoint open sets  $\{U_n\}_{n \geq 1}$  such that  $x_n \in U_n$ .

3) Let  $X$  be a topological space. Give the complete details to show that the set of real-valued continuous functions on  $X$  is a vector space.

4) Let  $\{X_\alpha\}$  be a family of  $T_2$  topological spaces. Let  $X = \prod X_\alpha$ . For a fixed  $\alpha$ , show that the canonical embedding of  $X_\alpha$  into  $X$  is a homeomorphism onto a closed subset of  $X$ .

5) Let  $X$  be a topological space. Show that a bijection  $f : X \rightarrow X$  is a homeomorphism if and only if  $f(\overline{A}) = \overline{f(A)}$

6) Give with details, examples of topological spaces  $X, Y$  such that  $X$  is homeomorphic to a subset of  $Y$ ,  $Y$  is homeomorphic to a subset of  $X$ , but  $X$  and  $Y$  are not homeomorphic.

7) Show that an open set in  $R^n$  has at most countably many components.

8) Let  $\tau$  be the usual topology on the set of real numbers  $R$ . Let  $\tau'$  be the topology with all sets of the form  $(b, \infty)$  as a subbasis. Show that for any topological space  $X$ ,  $f : X \rightarrow (R, \tau)$  is lower semicontinuous if and only if  $f : X \rightarrow (R, \tau')$  is continuous.